

Near-horizon microstates of the D1-D5-P black hole

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ABSTRACT: We construct probe solutions in the attractor background of the five-dimensional D1-D5-P black hole which represent near-horizon microstates in the limit of large D1-charge. These generalize the corresponding solutions considered by Gaiotto, Strominger and Yin for the 4-dimensional D0-D4 black hole. Using U-duality and a 4D-5D connection, we argue that the relevant configurations are bound states of D1-branes that have expanded through the Myers effect to form a Kaluza-Klein monopole wrapping the black hole horizon. We show that these branes experience a magnetic field on their moduli space, and that the degeneracy of lowest Landau levels reproduces the Bekenstein-Hawking entropy.

KEYWORDS: D-branes, Black Holes in String Theory, String Duality.

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1. Introduction and summary

The microscopic accounting of the Bekenstein-Hawking entropy of black holes is one of the successes of string theory [1, 2]. In most examples, such an accounting starts from the observation that the entropy (or supersymmetric index) doesn't depend on a coupling parameter and subsequently varying the coupling to a regime where a perturbative calculation is possible.

In an alternative, and in some sense more direct, approach, Gaiotto, Strominger and Yin have proposed to account for the black hole entropy from counting the supersymmetric bound states of the constituent D-branes as probes in the near-horizon geometry of the black hole [3]. We will refer to such probe configurations as ‘near-horizon microstates’ in what follows. In the case of the four-dimensional D0-D4 black hole and for large D0-charge, the relevant probe configurations are particular bound states of D0-branes [4, 5]. An attractive feature of the approach is that it gives insight into the physical mechanism behind the finite number of quantum states per unit horizon area of the black hole. The D0-brane probes experience a magnetic field on the internal space, which effectively divides the horizon into cells, each cell corresponding to a lowest Landau level ground state of the D0-brane probe mechanics.¹ A refined version of this approach was proposed in [7], where it was argued that the probe brane quantum mechanics arises from the moduli space

¹See [6] for an earlier application of Landau levels in black hole physics.

quantization of a multicentered solution carrying the same charges as the black hole. The mirror type IIB black hole case was considered in [8], and further related work appears in [9, 10].

In this work, we will generalize the near-horizon microstate approach to the case of the five-dimensional ‘D1-D5-P’ black hole in type IIB carrying wrapped D1-brane and D5-brane charges and momentum. We will construct near-horizon probes which are particular bound states of D1-branes and give an accounting of the entropy for large D1-charge. As in the four-dimensional examples, the degeneracy comes from counting lowest Landau levels in a magnetic field on the moduli space of the probe branes.

Our motivation for transposing the approach of [3] to the D1-D5-P black hole of [1, 2] is twofold. Firstly, in the D1-D5-P black hole there is a detailed understanding of the microscopic physics in terms of a dual conformal field theory (see [11] for a review). Hence we hope it will provide a good setting to address aspects of the near-horizon microstate approach which are not fully understood at present, such as incorporating subleading corrections to the entropy. A second motivation is that D1-D5 black holes provide the setting for another approach to black hole physics that was advocated by Lunin and Mathur [12]. In this approach, black hole ‘hair’ is represented by a family of nonsingular, horizonless classical supergravity solutions. For the D1-D5-P black hole, a subset of the microstate geometries is known [13]. It therefore seems a good starting point for trying to make contact between both approaches.

We will now summarize the contents of this paper. We start by considering a four-dimensional BPS black hole which carries 4 charges n, w, N, W with metric

$$ds_4^2 = -(H_n H_w H_N H_W)^{-1/2} dt^2 + (H_n H_w H_N H_W)^{1/2} (dr^2 + r^2 d\Omega_2^2) \quad (1.1)$$

and Bekenstein-Hawking entropy

$$S_4 = 2\pi\sqrt{nwNW}. \quad (1.2)$$

We will consider two different embeddings, related by a U-duality transformation, of such a black hole in toroidally compactified type II string theory.

In the first duality frame, referred to as ‘frame A’ and described in section 2.1, the charges correspond to D0-branes and D4-branes wrapping internal cycles. This is the setting of [3]. The near-horizon microstates are bound states of D0-branes that have expanded, through a form of the Myers effect [14], to form a D2-brane wrapping the horizon S^2 . We review this solution and its symmetry properties in section 2.2. Quantum mechanically, these D0-branes are described by a superconformal mechanics with $SU(1, 1|2)$ symmetry. Because they experience a magnetic field from the D4-branes in the background, as reviewed in section 2.3, their supersymmetric ground states have a large lowest Landau level degeneracy, which accounts for the entropy (1.2).

The second duality frame we will consider, referred to as ‘frame B’ and described in section 3.1, is an embedding as a ‘D1-D5-P-KK’ black hole [15, 16] in type IIB where the charges come from D1-branes, D5-branes, momentum and Kaluza-Klein (KK) monopole charge. The momentum and KK monopole charges produce nontrivial fibrations for two

internal circles, such that the near-horizon geometry has a component that is locally AdS_3 times a squashed three-sphere S^3/Z_W . Following the fate of the near-horizon microstate probes under the U-duality to frame B, we find that the relevant configuration is a bound state of D1-branes that has expanded to form a Kaluza-Klein monopole that wraps the horizon S^3/Z_W . We construct such a configuration explicitly as a solution of the Kaluza-Klein monopole worldvolume action [22–24] (reviewed in section 3.2) and show that it has the expected symmetry properties in section 3.3. In section 3.4 we show that the solution has a moduli space dynamics which includes a magnetic field, again reproducing the entropy from the counting of lowest Landau levels.

In section 4, we argue that similar Kaluza-Klein monopole solutions play the role of near-horizon microstates for the five dimensional black hole with D1-D5 and momentum charges and metric

$$ds_5^2 = -(H_n H_w H_N)^{-2/3} dt^2 + (H_n H_w H_N)^{1/3} (dr^2 + r^2 d\Omega_3^2). \quad (1.3)$$

The argument uses a version of the 4D-5D connection [17–19]: by decompactifying one of the internal circles, the D1-D5-P-KK black hole considered above lifts to a five-dimensional D1-D5-P black hole in the center of an orbifold space R^4/Z_W . Since the size of the decompactified circle is a fixed scalar, the near-horizon geometry does not change under the decompactification, and the relevant near-horizon probes are again bound states of D1-branes expanded to form a Kaluza-Klein monopole. The special case $W = 1$ gives the black hole in flat space (1.3), and counting the lowest Landau level degeneracy reproduces its entropy

$$S_5 = 2\pi\sqrt{nwN}. \quad (1.4)$$

2. The D0-D4 black hole in type IIA

In this section we review some aspects of the near-horizon microstate approach for the four-dimensional D0-D4 black hole in type IIA [3]. For simplicity, we consider a toroidal $N = 8$ compactification throughout this paper, but the arguments could be repeated for the case of a $N = 4$ compactification on $T^2 \times K_3$.

2.1 Background

We first consider type IIA compactified on a rectangular six-torus T^6 which we regard as a product of two circles S^1, \tilde{S}^2 and two tori T^2, \tilde{T}^2 . We embed the 4-dimensional 1/8 BPS black hole of (1.1) with charges n, w, N, W as a configuration consisting of D0-branes and D4-branes wrapping internal cycles as follows:

$$\begin{aligned}
 \text{(frame A)} \quad & n \text{ D0-branes} \\
 & w \text{ D4-branes wrapped on } T^2 \times \tilde{T}^2 \\
 & N \text{ D4-branes wrapped on } S^1 \times \tilde{S}^1 \times T^2 \\
 & W \text{ D4-branes wrapped on } S^1 \times \tilde{S}^1 \times \tilde{T}^2
 \end{aligned}$$

We will refer to this string theory embedding as ‘duality frame A’ in what follows. The 10-dimensional string metric is

$$ds_{10}^2 = -(H_n H_w H_N H_W)^{-1/2} dt^2 + (H_n H_w H_N H_W)^{1/2} (dr^2 + r^2 d\Omega_2^2) \\ + \frac{1}{4} \left(\frac{H_n H_w}{H_N H_W} \right)^{1/2} \left(R^2 dx^2 + \tilde{R}^2 d\tilde{x}^2 \right) + \left(\frac{H_n H_W}{H_w H_N} \right)^{1/2} ds_{T^2}^2 + \left(\frac{H_n H_N}{H_w H_W} \right)^{1/2} ds_{\tilde{T}^2}^2$$

Here, R and \tilde{R} denote the radii of S^1 and \tilde{S}^1 on which we have chosen coordinates x, \tilde{x} with periodicity 4π . We will work in the units $2\pi\sqrt{\alpha'} = 1$, where the fundamental string and D-brane charges take the value 2π . The harmonic functions are given in terms of the quantized charges as

$$\begin{aligned} H_n &= 1 + \frac{g_\infty}{4\pi V_{T^6}} \frac{n}{r} & H_w &= 1 + \frac{g_\infty}{4\pi(2\pi R)(2\pi\tilde{R})} \frac{w}{r} \\ H_N &= 1 + \frac{g_\infty}{4\pi V_{\tilde{T}^2}} \frac{N}{r} & H_W &= 1 + \frac{g_\infty}{4\pi V_{T^2}} \frac{W}{r} \end{aligned} \quad (2.1)$$

The dilaton and RR gauge fields are given by

$$e^\Phi = g_\infty H_n^{3/4} (H_w H_N H_W)^{-1/4} \\ C^{(1)} = -\frac{1}{g_\infty} \left(\frac{1}{H_n} - 1 \right) dt; \quad C^{(3)} = -\frac{1}{4\pi} \cos\theta d\phi \wedge [w \omega_{S^1 \times \tilde{S}^1} + W \omega_{T^2} + N \omega_{\tilde{T}^2}]$$

Here, the $\omega_{\mathcal{M}_2}$ are normalized volume forms satisfying $\int_{\mathcal{M}_2} \omega_{\mathcal{M}_2} = 1$ and g_∞ is the value of the string coupling at infinity.

We will be interested in the near-horizon scaling limit of this geometry, which is obtained by temporarily restoring α' factors and taking

$$\alpha' \rightarrow 0; \quad \frac{r}{\alpha'}, \frac{R}{\sqrt{\alpha'}}, \frac{\tilde{R}}{\sqrt{\alpha'}}, \frac{V_{T^2}}{\alpha'}, \frac{V_{\tilde{T}^2}}{\alpha'} \text{ fixed} \quad (2.2)$$

In this limit, the above background reduces to an $AdS_2 \times S^2 \times T^6$ attractor geometry where the $AdS_2 \times S^2$ radius and the volumes of the tori $S^1 \times \tilde{S}^1$, T^2 and \tilde{T}^2 are fixed in terms of the charges. Performing a coordinate change to global AdS_2 coordinates

$$r = l_A (\cosh \chi \cos \tau + \sinh \chi); \quad t = \frac{l_A^2}{r} \cosh \chi \sin \tau \quad (2.3)$$

as well as a gauge transformation on $C^{(1)}$, we obtain the near-horizon geometry

$$ds_{10}^2 = l_A^2 [-\cosh^2 \chi d\tau^2 + d\chi^2 + d\theta^2 + \sin^2 \theta d\phi^2] \\ + \sqrt{\frac{n}{wNW}} \left[\frac{w}{16\pi^2} \left(\frac{R}{\tilde{R}} dx^2 + \frac{\tilde{R}}{R} d\tilde{x}^2 \right) + \frac{W}{V_{T^2}} ds_{T^2}^2 + \frac{N}{V_{\tilde{T}^2}} ds_{\tilde{T}^2}^2 \right] \\ C^{(1)} = -\frac{1}{4\pi} \sqrt{\frac{wNW}{n}} \sinh \chi d\tau; \quad C^{(3)} = -\frac{1}{4\pi} \cos\theta d\phi [w \omega_{S^1 \times \tilde{S}^1} + W \omega_{T^2} + N \omega_{\tilde{T}^2}] \quad (2.4)$$

The $AdS_2 \times S^2$ radius l_A is given by

$$l_A = \frac{g}{4\pi} \sqrt{\frac{wNW}{n}}. \quad (2.5)$$

Here, g denotes the value of the string coupling in the near-horizon region. The supergravity description is reliable as long as $g \ll 1$ and $l_A \gg 1$.

The near-horizon geometry preserves 8 Killing spinors, which combine with the $SL(2, R)$ isometry group of AdS_2 and the $SO(3)$ symmetry of S^2 into an $SU(1, 1|2)$ superisometry group. The Killing vectors generating the $SL(2, R)$ component are given by

$$\begin{aligned} l_0 &= \partial_\tau \\ l_\pm &= e^{\pm i\tau} [\tanh \chi \partial_\tau \mp i \partial_\chi] \end{aligned} \tag{2.6}$$

2.2 Horizon-wrapping membranes and their symmetries

The near-horizon microstates that capture the entropy of the D0-D4 black hole at large D0-charge are particular bound states of D0-branes that have expanded to form a D2-brane, wrapping the horizon S^2 , through a form of the Myers effect. These can be described as noncommutative solutions in the multi-D0-brane action or, alternatively, as solutions of the D2-brane action with D0-brane charge dissolved on the worldvolume. We will here focus on the latter description and describe the probe solution and its properties in more detail.

We consider a D2-brane probe in the background (2.4), wrapping the horizon S^2 , and choose a static gauge such that the worldvolume coordinates coincide with τ, θ, ϕ . Turning on worldvolume flux \mathcal{F} on S^2 induces Q units of D0-brane charge:

$$\mathcal{F} = \frac{Q}{4\pi} \sin \theta d\theta d\phi.$$

Dimensionally reducing over the two-sphere, the Lagrangian describing the motion of such a brane reads

$$\mathcal{L} = -Ml_A \left[\sqrt{1 + \rho^2} \sqrt{\cosh^2 \chi - \dot{\chi}^2} + \sinh \chi \right]. \tag{2.7}$$

We have restricted attention to a D2-brane that is static on the T^6 for the time being. The parameters M and ρ correspond to the mass of the wrapped D2-brane and the induced D0-brane charge density on S^2 respectively:

$$\begin{aligned} M &= 4\pi l_A^2 T_{D_2} = \frac{g}{2} \frac{wNW}{n} \\ \rho &= \frac{Q}{4\pi l_A^2} = \frac{4\pi Q}{g^2} \frac{n}{wNW}. \end{aligned} \tag{2.8}$$

The isometries (2.6) of the background act as symmetries on the D2-brane worldvolume and lead to Noether charges L_0, L_\pm , where L_0 is the canonical Hamiltonian obtained from (2.7). They are given by

$$L_0 = \cosh \chi \sqrt{P_\chi^2 + (Ml_A)^2(1 + \rho^2)} + Ml_A \rho \sinh \chi \tag{2.9}$$

$$L_\pm = e^{\pm i\tau} \left[\tanh \chi L_0 \pm iP_\chi + \frac{Ml_A \rho}{\cosh \chi} \right] \tag{2.10}$$

These expressions are derived by varying the D2-brane action before gauge-fixing the worldvolume time coordinate, and the last term in (2.10) arises because the Wess-Zumino term

is only invariant up a total derivative. From (2.7) or (2.9) we see that there is a static solution where the brane is located at

$$\sinh \chi = -\rho.$$

The Noether charges (2.10) evaluated on this solution are [10]

$$L_0 = Ml_A \tag{2.11}$$

$$L_{\pm} = 0 \tag{2.12}$$

Hence the solution is ‘primary’ and, in addition, invariant under conformal boosts:

$$K = L_+ + L_- = 0. \tag{2.13}$$

The supersymmetry properties of this solution were analyzed in [4], where it was shown to preserve half of the near-horizon supersymmetries. Since it is static with respect to global time τ instead of Poincaré time t , it breaks all of the Poincaré supersymmetries that extend to the asymptotically flat region. Such branes are necessarily bound to the near-horizon region and have an energy barrier preventing them to escape to asymptotic infinity.

2.3 Landau levels on moduli space and microstate counting

A important property of the horizon-wrapping membranes is that they experience a magnetic field, induced by the D4-brane charges in the background, on their moduli space. Due to this fact, the supersymmetric ground states in the quantized theory have a large lowest Landau level degeneracy which accounts for the black hole entropy, as we shall presently review.

The energy of the probe solutions is independent of the position of the probe on T^6 . Hence these positions are bosonic moduli of our solution and the moduli space \mathcal{M} equals T^6 . The moduli space mechanics is that of a particle moving on T^6 , with kinetic terms come from expanding the Born-Infeld action. The particle also couples (with charge 2π) to a magnetic field which comes from the Wess-Zumino coupling $\int C^{(3)}$ to the D4-branes in the background. From the expression (2.4), we see that this term gives rise to a magnetic field with field strength

$$F_{\mathcal{M}} = w \omega_{S^1 \times \tilde{S}^1} + W \omega_{T^2} + N \omega_{\tilde{T}^2} \tag{2.14}$$

Hence the particle moves in a magnetic field with w units of flux through $S^1 \times \tilde{S}^1$, W units of flux through T^2 and N units of flux through \tilde{T}^2 .

The full quantum mechanical theory describing the low-energy dynamics of the horizon-wrapping membrane was constructed in [5]. The theory realizes the superconformal algebra $SU(1,1|2)$ with a central charge Ml_A , with M given in (2.8). The corresponding BPS bound is saturated by chiral primary states which satisfy (2.11). In [3], it was shown that these chiral primaries are in one-to-one correspondence with lowest Landau levels in the magnetic field $F_{\mathcal{M}}$. More precisely, by using a standard representation for the fermionic zero modes on differential forms in moduli space, it was shown that chiral primaries are represented by harmonic forms h with respect to a covariant derivative \bar{D} :

$$\bar{D}h = \bar{D}^\dagger h = 0 \tag{2.15}$$

where $\bar{D} = \bar{\partial} + \bar{A}_{\mathcal{M}}$ and $\bar{A}_{\mathcal{M}}$ is a holomorphic connection for $F_{\mathcal{M}}$. Even forms represent bosons while odd forms represent fermions. The number of solutions to (2.15) could be computed by index theory as in [3] but, in the simple toroidal case we are considering, one can also enumerate the solutions explicitly. On a single two-torus \mathcal{N} units of magnetic flux, there are harmonic (0,0) forms Ψ_J and harmonic (1,0) forms $\Psi_J dz$. The Ψ_J , where J is an angular momentum quantum number running from 0 to $\mathcal{N} - 1$, are lowest Landau level wavefunctions on the torus whose explicit form can be found e.g. in [20]. In the case at hand, we have a product space of three tori with magnetic field, and we get a total of $4wNW$ bosonic and $4wNW$ fermionic solutions.

It is important to observe that the number of chiral primaries does not depend on the background D0-charge n , and for the purpose of state counting we can imagine all the D0-charge to be carried by the probe branes. The total D0-charge n can then be divided in many ways into clusters of D0-branes forming D2 bound states considered above. Counting such multiparticle chiral primaries is equivalent to counting the degeneracy in a CFT with $4wNW$ bosons and $4wNW$ fermions at level n . The central charge c is $6wNW$ and the degeneracy $D(n)$ at large n is given by the Cardy formula

$$\ln D(n) = 2\pi\sqrt{nc/6} = 2\pi\sqrt{nwNW}$$

in agreement with the macroscopic entropy (1.2).

We end this section with some remarks:

- The above picture, where we counted bound states of D0-branes, was valid for black holes which are ‘mostly made up out of D0-branes’ where the D0-charge n is parametrically larger than the D4-brane charges. As a result, the calculation does not capture corrections to the entropy subleading in n , which can easily be seen from the fact that the full D0-brane partition function is not U-duality invariant.
- We should also remark that the above calculation is not a fully controlled approximation: in the limit of parametrically large n where we did the microstate counting, we see from (2.5) that it is not possible to keep both the string coupling small while keeping the S^2 radius large in string units. Strong coupling problems of this type are typically resolved by going to a different duality frame where the approximations are under control; this will be the case for the U-dual description we will consider in the next section.
- We also observe that there is a similar picture of near-horizon microstates for black holes which are ‘mostly made up out of D4 branes’, i.e. where one of the D4-charges is parametrically larger than the other charges. This situation is T-dual to the case considered above. The near-horizon probe solutions are now D6-branes which wrap the horizon S^2 and a four cycle, with worldvolume flux on the S^2 . The moduli space is again a T^6 , spanned by two transverse directions and four Wilson lines, and magnetic fields on moduli space are produced by the couplings $\int C^{(3)} \wedge \mathcal{F} \wedge \mathcal{F}$ and $\int C^{(7)}$.

IIA		IIB		IIB		IIB
D0		D3		D3		D1(S^1)
$D4(T^2, \tilde{T}^2)$	T (\tilde{S}^1, T^2)	D3	S	D3	T (S^1, \tilde{S}^1, T^2)	$D5(S^1, T^2, \tilde{T}^2)$
$D4(S^1, \tilde{S}^1, T^2)$	→	D1	→	F1	→	P(S^1)
$D4(S^1, \tilde{S}^1, T^2)$		D5		NS5		KK($S^1, \tilde{S}_{\text{TN}}^1, T^2, \tilde{T}^2$)

Table 1: Duality transformations relating frame A to frame B.

3. U-duality and the D1-D5-P-KK black hole

3.1 Duality chain

In this section we will describe an embedding of the 4-dimensional black hole (1.1) as a ‘D1-D5-P-KK’ black hole solution in type IIB, with the charges n, w, N, W corresponding to D1, D5, momentum and Kaluza-Klein monopole charge respectively. This solution can be obtained from the one in duality frame A in the previous section through a U-duality transformation of the form TST as illustrated in Table 1. The final configuration, which we will refer to as ‘duality frame B’, is

- (frame B)
- n D1-branes wrapped on S^1
 - w D5-branes wrapped on $S^1 \times T^2 \times \tilde{T}^2$
 - N units of momentum on S^1
 - W KK-monopoles, Taub-NUT direction \tilde{S}^1 , wrapped on $S^1 \times T^2 \times \tilde{T}^2$

In this duality frame, the S^1 and \tilde{S}^1 circles are fibred nontrivially due to the momentum and KK monopole charges. The 10-dimensional metric is

$$\begin{aligned}
 ds_{10}^2 = & (H_n H_w)^{-1/2} \left[-\frac{1}{H_N} dt^2 + H_N \left(\frac{R}{2} dx - (1/H_N - 1) dt \right)^2 \right] \\
 & + (H_n H_w)^{1/2} \left[H_W (dr^2 + r^2 d\Omega_2^2) + \frac{\tilde{R}^2}{4H_W} (d\tilde{x} - W \cos \theta d\phi)^2 \right] + (H_n/H_w)^{1/2} (ds_{T^2}^2 + ds_{\tilde{T}^2}^2)
 \end{aligned} \tag{3.1}$$

The harmonic functions are now given by

$$\begin{aligned}
 H_n &= 1 + \frac{g_\infty}{4\pi(2\pi\tilde{R})V_{T^2}V_{\tilde{T}^2}} \frac{n}{r} & H_w &= 1 + \frac{g_\infty}{4\pi(2\pi\tilde{R})} \frac{w}{r} \\
 H_N &= 1 + \frac{g_\infty^2}{4\pi(2\pi\tilde{R})V_{T^6}} \frac{N}{r} & H_W &= 1 + \frac{\tilde{R}W}{2} \frac{1}{r}.
 \end{aligned} \tag{3.2}$$

For the dilaton and RR fields, we have

$$\begin{aligned}
 e^\Phi &= g_\infty H_n^{1/2} H_w^{-1/2} \\
 C^{(2)} &= -\frac{R}{2g_\infty} (1/H_n - 1) dt \wedge dx - \frac{w}{16\pi^2} \cos \theta d\phi \wedge d\tilde{x}
 \end{aligned} \tag{3.3}$$

We now take a near-horizon scaling limit that matches the one we considered in frame A (2.2), as well as a rescaling of t and a coordinate change (2.3). One of the gauge transformations we performed in frame A now becomes a shift of the coordinate x , leading

IIA	T (\tilde{S}^1, T^2)	IIB	S	IIB	T (S^1, \tilde{S}^1, T^2)	IIB
D2(S^2)	→	D5	→	NS5	→	KK ($S^1_{\text{TN}}, S^3/Z_W, T^2$)

Table 2: Transformation of D-brane probes from duality frame A to frame B.

to a new variable by x' with period 4π . We obtain an attractor geometry where, this time, the fixed moduli are the radii of S^1 and \tilde{S}^1 and the volume of the 4-torus $T^2 \times \tilde{T}^2$:

$$\begin{aligned}
 ds_{10}^2 &= l_B^2 \left[-\cosh^2 \chi d\tau^2 + dx'^2 + d\theta^2 + \sin^2 \theta d\phi^2 \right] + k^2 (dx' + A)^2 + \tilde{k}^2 (d\tilde{x} + \tilde{A})^2 \\
 &\quad + \sqrt{\frac{n}{w}} \frac{ds_{T^2}^2 + ds_{\tilde{T}^2}^2}{\sqrt{V_{T^2} V_{\tilde{T}^2}}} \\
 C^{(2)} &= \frac{1}{g} (k^2 A \wedge dx' + \tilde{k}^2 \tilde{A} \wedge d\tilde{x})
 \end{aligned} \tag{3.4}$$

with the Kaluza-Klein scalars k, \tilde{k} and one-forms A, \tilde{A} given by

$$k^2 = l_B^2 \frac{N}{nwW}; \quad \tilde{k}^2 = \frac{l_B^2}{W^2}; \quad A = -\sqrt{\frac{nwW}{N}} \sinh \chi d\tau; \quad \tilde{A} = -W \cos \theta d\phi. \tag{3.5}$$

The radius l_B is given by

$$l_B = \frac{\sqrt{g}}{4\pi} \sqrt{wW} \tag{3.6}$$

where g is the string coupling in the near-horizon region. We can trust the supergravity description as long as $g \ll 1$ and $gwW \gg 1$.

In (3.4), the circles S^1 and \tilde{S}^1 are ‘Hopf’-fibered over AdS_2 and S^2 respectively so as to form a space which is locally $AdS_3 \times S^3$ with curvature radii $l_{AdS_3} = l_{S^3} = 2l_B$. Due to the compactness of x' and the KK monopole charge W (when $W > 1$) however, the space is not globally $AdS_3 \times S^3$, but rather the product of ‘squashed’ AdS_3 with the squashed three-sphere S^3/Z_W [25]. The squashing preserves the left-moving isometry group $SL(2, R)_L \times SO(3)_L$, combining with fermionic generators into an $SU(1, 1|2)$ supergroup, while the right-moving $SL(2, R)_R \times SO(3)_R$ is broken down to two $U(1)$ ’s which act as translations of x' and \tilde{x} . Hence we find the same super-isometry group as in frame A, as of course we should. The Killing vectors generating $SL(2, R)_L$ are given by

$$\begin{aligned}
 l_0 &= \partial_\tau \\
 l_\pm &= e^{\pm i\tau} \left[\tanh \chi \partial_\tau \mp i \partial_\chi - \frac{1}{\cosh \chi} \partial_{x'} \right]
 \end{aligned} \tag{3.7}$$

Having discussed the duality transformation relating the D0-D4 black hole to the D1-D5-P-KK black hole, we will now apply the same dualities to the near-horizon microstate probes of frame A in order to find microstate probes in frame B. Under the U-duality transforming frame A into frame B, a D2-brane wrapping S^2 in frame A transforms as indicated in table 2. The first arrow is essentially mirror symmetry, and leads to the near-horizon probe picture discussed in [8]. The final probe configuration in frame B is a Kaluza-Klein (KK) monopole wrapped on the near-horizon S^3/Z_W as well as on T^2 and whose Taub-NUT direction is along S^1 . The configuration in frame A also carried D0-brane

charge Q induced by worldvolume flux on S^2 . The KK-monopole probe in frame B similarly has an appropriate worldvolume field turned on so as to induce D1-brane charge along S^1 . We will construct such a solution explicitly in section 3.3 from the worldvolume action of the KK monopole, and check that it carries the same Noether charges as its counterpart in frame A. In section 3.4, we will show that its moduli space mechanics includes magnetic fields of the correct magnitude.

3.2 The worldvolume action for a KK monopole

Let us first review some facts about the effective worldvolume description of KK-monopoles. The worldvolume theory of a KK-monopole in type IIB is a $(2, 0)$ theory in 5+1 dimensions, and the collective coordinates organize themselves into a tensor multiplet [21]. The worldvolume dynamics, which is determined by dualities relating the KK-monopole to other branes [23], cannot be captured by a covariant action due to the selfduality condition on the tensor field. To avoid this difficulty, we will make use of the observation of [24] that, after dimensional reduction to 4+1 dimensions, the tensor multiplet reduces to a $(1, 1)$ vector multiplet which can be described by a covariant action. Therefore, if we consider KK monopole which is wrapped on at least one compact direction, we can use a dimensionally reduced 4+1 dimensional action, which can be obtained by T-dualizing the action for the type IIA KK-monopole constructed in [22] along a worldvolume direction.

The wrapped KK-monopole action thus obtained can be formulated in spacetimes which have two compact isometry directions: the first one, which we will call k^μ , denotes the Taub-NUT circle of the monopole, while the second one, \tilde{k}^μ , is the circle on which the monopole is wrapped. The field content is summarized in the table 3 and consists of three scalars X^i , describing transverse fluctuations, two zero-forms $\omega^{(0)}, \tilde{\omega}^{(0)}$ (with field strengths $\mathcal{G}^{(1)}, \tilde{\mathcal{G}}^{(1)}$) which source fundamental and D-string charge along the Taub-NUT direction k^μ , and a one-form $\mathcal{A}^{(1)}$ (with field strength $\mathcal{F}^{(2)}$).

worldvolume field	field strength
X^i	-
$\omega^{(0)}$	$\mathcal{G}^{(1)}$
$\tilde{\omega}^{(0)}$	$\tilde{\mathcal{G}}^{(1)}$
$\mathcal{A}^{(1)}$	$\mathcal{F}^{(2)}$

We now place a KK-monopole probe in the background (3.4), taking the Taub-NUT direction to be along the circle S^1 : $k^\mu = (\partial_{x'})^\mu$. It will be convenient to choose the wrapping direction to be along the \tilde{S}^1 circle so that $\tilde{k}^\mu = (\partial_{\tilde{x}})^\mu$. The wrapped KK-monopole action in this background

Table 3: Worldvolume fields appearing in the KK monopole action.

$$\begin{aligned}
 S = & -\tau_{KK} \int d^5\sigma k^2 \tilde{k} e^{-2\Phi} \sqrt{-\det(P[\tilde{G}]_{ab} + k^{-2} \mathcal{G}_a^{(1)} \mathcal{G}_b^{(1)} + e^{2\Phi} k^{-2} \tilde{\mathcal{G}}_a^{(1)} \tilde{\mathcal{G}}_b^{(1)} - e^\Phi k^{-1} \tilde{k}^{-1} \mathcal{F}_{ab}^{(2)})} \\
 & + \tau_{KK} \int \left[P[i_{\tilde{k}} i_k N^{(7)}] + \frac{1}{2} P[\tilde{A}] \wedge \mathcal{F}^{(2)} \wedge \mathcal{F}^{(2)} + P[A] \wedge \mathcal{F}^{(2)} \wedge \mathcal{G}^{(1)} \wedge \tilde{\mathcal{G}}^{(1)} \right] \quad (3.8)
 \end{aligned}$$

Here, we have denoted pullbacks by $P[\dots]$, while $\tilde{G}_{\mu\nu}$ is the metric on the 8-dimensional base space over which the S^1 and \tilde{S}^1 circles are fibered:

$$\tilde{G}_{\mu\nu} = G_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - \frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^2}.$$

The 7-form $N^{(7)}$ is a gauge potential for the 8-form field strength which is dual to the KK one-form A , with

$$i_{\tilde{k}} i_k N^{(7)} = \frac{2\pi P}{\tau_{KK}} \omega_{T^2} \wedge \omega_{S^2} \wedge \frac{\tilde{y}^1 d\tilde{y}^2}{V_{\tilde{T}^2}}.$$

The worldvolume field strengths entering in (3.8) are

$$\begin{aligned} \mathcal{G}^{(1)} &= d\omega^{(0)} \\ \tilde{\mathcal{G}}^{(1)} &= d\tilde{\omega}^{(0)} + P[i_{\tilde{k}} C^{(2)}] \end{aligned} \tag{3.9}$$

$$\mathcal{F}^{(2)} = dA^{(1)}/(4\pi)^2 - P[i_{\tilde{k}} C^{(2)}] \wedge \mathcal{G}^{(1)}. \tag{3.10}$$

The normalization constant τ_{KK} takes the value $\tau_{KK} = 8(2\pi)^4$ in our units, and is related to the physical tension T_{KK} as $T_{KK} = \tau_{KK} k^2 \tilde{k}/g^2$.

3.3 Horizon-wrapping KK-monopoles and their symmetries

We shall now explicitly construct the KK-monopole probe solutions which will play the role of near-horizon microstates of the D1-D5-P-KK black hole, and show that they have the same symmetry properties as their counterparts in frame A. Choosing coordinates y^1, y^2 on T^2 and \tilde{y}^1, \tilde{y}^2 on \tilde{T}^2 , we will work in a static gauge where the worldvolume coordinates are identified with $\tau, \theta, \phi, y^1, y^2$. As discussed in section 3.1, we want to consider a KK-monopole that carries induced D1-brane charge by turning on appropriate worldvolume fields. Such a solution can be interpreted as a bound state of D1-branes that have expanded into a KK monopole through the Myers effect. From the relation (3.9) we see that turning on time-dependent $\tilde{\omega}^{(0)}$ sources D1-brane charge along S^1 , so that the conserved momentum conjugate to $\tilde{\omega}^{(0)}$ will be proportional to the induced D1-brane charge.

The Lagrangian for such a KK monopole, dimensionally reduced over $S^2 \times T^2$, is given by

$$\mathcal{L} = -Ml_B \sqrt{\cosh^2 \chi - (\beta \dot{\tilde{\omega}}^{(0)} - \sinh \chi)^2} \tag{3.11}$$

We have restricted attention to a static probe on \tilde{T}^2 with constant gauge fields $\omega^{(0)}, \mathcal{A}^{(1)}$. The constants M (representing the mass of the wrapped KK-monopole) and β are given by

$$M = T_{KK} 4\pi l_B^2 \sqrt{\frac{n}{w}} \sqrt{\frac{V_{T^2}}{V_{\tilde{T}^2}}}; \quad \beta = \frac{g}{k^2} \left(\frac{N}{nwW} \right)^{1/2} \tag{3.12}$$

The momentum conjugate to $\tilde{\omega}^{(0)}$ is related to the induced D1-brane charge Q as

$$P_{\tilde{\omega}^{(0)}} = 2\pi Q. \tag{3.13}$$

We observe from the form of (3.11) that the background fields in (3.4) have conspired to produce a Lagrangian describing the motion of a particle on a locally AdS_3 space, with $\tilde{\omega}^{(0)}$ playing the role of the Hopf fibre coordinate. Hence we can easily find the $SL(2, R)_L$ Noether charges L_0, L_{\pm} from (3.7):

$$L_0 = \cosh \chi \sqrt{P_{\chi}^2 + (Ml_B)^2 (1 + \rho^2)} + Ml_B \rho \sinh \chi \tag{3.14}$$

$$L_{\pm} = e^{\pm i\tau} \left[\tanh \chi L_0 \pm iP_{\chi} + \frac{Ml_B \rho}{\cosh \chi} \right]. \tag{3.15}$$

Here, L_0 is the canonical Hamiltonian obtained from (3.11). To derive these expressions, we have used (3.13) and defined ρ as $\rho = 2\pi Q/Ml_B\beta$. With these definitions, the Noether charges take precisely the same form as in frame A (2.10). Again, there is a static solution which now represents a wrapped KK-monopole located at

$$\sinh \chi = -\rho. \tag{3.16}$$

It has Noether charges $L_0 = Ml_B, L_{\pm} = 0$ and is the sought-after configuration U-dual to the horizon-wrapping membrane in frame A.

As the κ -symmetry transformations of the KK-monopole action are not known at present, it is not possible to directly verify the preserved supersymmetries of the solution. U-duality predicts that it should have the same supersymmetry properties as its counterpart in frame A, namely preserving half of the supersymmetries while breaking all Poincaré supersymmetries.

3.4 Moduli space dynamics and state counting

We will now consider the moduli space dynamics of the probe solution considered above. As in frame A, we will see that our probes experience a magnetic field on moduli space and that the lowest Landau level degeneracy accounts for the Bekenstein-Hawking entropy. Due to the complexity of the action (3.8), the analysis will be more involved than in frame A. We will see that the magnetic field on moduli space now arises both from Born-Infeld and Wess-Zumino terms in the action (3.8).

The energy of the above solutions is independent of the constant values of the world-volume fields $\omega^{(0)}, \mathcal{A}^{(1)}, \tilde{y}^1, \tilde{y}^2$, hence these will give rise to the moduli of the solution. The moduli space mechanics is obtained in a standard manner by expanding the action around the solution (3.16) to quadratic order in the fields $\omega^{(0)}, \mathcal{A}^{(1)}, \tilde{y}^1, \tilde{y}^2$ and dimensionally reducing to 0+1 dimensions. The quadratic action is

$$S_2 = -\tau_{KK} \int \left[k^2 \tilde{k} e^{-2\Phi} \sqrt{\frac{n}{wV_{T^2}V_{\tilde{T}^2}}} (d\tilde{y}^1 \wedge \star d\tilde{y}^1 + d\tilde{y}^2 \wedge \star d\tilde{y}^2) + \frac{1}{2} \tilde{k} e^{-2\Phi} d\omega^{(0)} \wedge \star d\omega^{(0)} + \frac{1}{4\tilde{k}} \mathcal{F}^{(2)} \wedge \star \mathcal{F}^{(2)} \right] + \tau_{KK} \int \left[P[i_{\tilde{k}} i_{\tilde{k}} N^{(7)}] + \frac{1}{2} P[\tilde{A}] \mathcal{F}^{(2)} \wedge \mathcal{F}^{(2)} \right]. \tag{3.17}$$

where

$$\mathcal{F}^{(2)} = \frac{dA^{(1)}}{(4\pi)^2} + \frac{\tilde{k}^2}{g} P[\tilde{A}] \wedge d\omega^{(0)}.$$

The Hodge \star is to be taken with respect to the worldvolume metric

$$ds_{wv}^2 = l_B^2 (-d\tau^2 + d\theta^2 + \sin^2 \theta d\phi^2) + \sqrt{\frac{n}{wV_{T^2}V_{\tilde{T}^2}}} ((dy^1)^2 + (dy^2)^2).$$

Note that the kinetic terms in (3.17) are not diagonal due to the mixing between $\mathcal{A}^{(1)}$ and $\omega^{(0)}$. We will now perform the dimensional reduction along with field redefinitions so as to obtain diagonal kinetic terms in 0+1 dimensions.

First, we dimensionally reduce over the T^2 directions y^1, y^2 to three dimensions. The reduction of the field $\mathcal{A}^{(1)}$ gives two Wilson lines w^1, w^2 from the components along y^1, y^2

and a gauge field $\mathcal{A}^{(1)}$ with curvature $\mathcal{F}^{(2)} = d\mathcal{A}^{(1)}$. Next, we dualize $\mathcal{A}^{(1)}$ into a zero-form $\tilde{\mathcal{A}}^{(0)}$ by adding a term $\int d\tilde{\mathcal{A}}^{(0)} \wedge \mathcal{F}^{(2)}$ to the action and integrating over $\mathcal{F}^{(2)}$. This produces a kinetic term for $\tilde{\mathcal{A}}^{(0)}$ and, because of the mixing terms in (3.17), an additional Wess-Zumino type term $w/W \int P[\tilde{A}] \wedge d\omega^{(0)} \wedge d\tilde{\mathcal{A}}^{(0)}$. After partial integrations and a further reduction over S^2 , we obtain a particle action on a rectangular six-torus with coordinates $\omega^{(0)}, \tilde{\mathcal{A}}^{(0)}, w^1, w^2, \tilde{y}^1, \tilde{y}^2$ in a magnetic field. The magnetic field strength is

$$F_{\mathcal{M}} = w \, 2d\omega^{(0)} \wedge d\tilde{\mathcal{A}}^{(0)} + WV_{T^2}dw^1 \wedge dw^2 + \frac{N}{V_{\tilde{T}^2}}d\tilde{y}^1 \wedge d\tilde{y}^2. \quad (3.18)$$

Taking into account the periodicities of the moduli space coordinates,² we again find three tori with w, W and N units of magnetic flux respectively. As explained in section 2.3, the counting of chiral primary states of the KK-monopole theory reduces to counting lowest Landau level degeneracies and reproduces the entropy (1.2).

4. 4D-5D connection and the D1-D5-P black hole

We now discuss the relevance of the probe solutions constructed above to the description of near-horizon microstates in five-dimensional black holes. The reason for this is that the D1-D5-P-KK background (3.3) lends itself to a version of the 4D-5D connection which was also at the basis of the earlier work [15, 16, 26] and which we will outline here.

So far, we have assumed the radius \tilde{R} of the \tilde{S}^1 circle to be small compared to the size l_B of the black hole. In this regime, the appropriate picture is that of a black hole in four dimensions (1.1). We now vary the radius to the regime where $\tilde{R} \gg l_B$, where the geometry effectively looks five-dimensional and describes a five-dimensional black hole with D1-D5-P charges (n, w, N) , placed at the center of a Taub-NUT space with NUT charge W . The relevant limit to describe this five-dimensional regime is to take the decompactification limit keeping $\tilde{R}r$ fixed:

$$\tilde{R} \rightarrow \infty; \quad \tilde{r}^2 \equiv 2\tilde{R}r \text{ fixed.} \quad (4.1)$$

The background (3.3) becomes

$$ds_{10}^2 = (H_n H_w)^{-1/2} \left[-\frac{1}{H_N} dt^2 + H_N \left(\frac{R}{2} dx - (1/H_N - 1) dt \right)^2 \right] \quad (4.2)$$

$$+ W (H_n H_w)^{1/2} \left[d\tilde{r}^2 + \frac{1}{4} \tilde{r}^2 \left(d\Omega_2^2 + \left(\frac{1}{W} d\tilde{x} - \cos \theta d\phi \right)^2 \right) \right] + (H_n/H_w)^{1/2} (ds_{T^2}^2 + ds_{\tilde{T}^2}^2)$$

and the harmonic functions are the correct ones for objects in five noncompact dimensions:

$$H_n = 1 + \frac{g_\infty}{4\pi^2 V_{T^2} V_{\tilde{T}^2}} \frac{n}{\tilde{r}^2} \quad H_w = 1 + \frac{g_\infty}{4\pi^2} \frac{w}{\tilde{r}^2}$$

$$H_N = 1 + \frac{g_\infty^2}{4\pi^2 (2\pi R)^2 V_{T^2} V_{\tilde{T}^2}} \frac{N}{\tilde{r}^2}. \quad (4.3)$$

²The periodicity of $\omega^{(0)}$ is $1/4\pi$, while $\mathcal{A}^{(0)}$ has period 2π , and w^1, w^2 have the inverse periodicities of y^1, y^1 .

For the dilaton and RR fields, we have

$$e^\Phi = g_\infty H_n^{1/2} H_w^{-1/2}$$

$$C^{(2)} = -\frac{R}{2g_\infty} (1/H_n - 1) dt \wedge dx - \frac{w}{16\pi^2} \cos \theta d\phi \wedge d\tilde{x} \quad (4.4)$$

In five dimensions, the metric in the Einstein frame reads

$$ds_5^2 = -(H_n H_w H_N)^{-2/3} dt^2 + (H_n H_w H_N)^{1/3} W \left[d\tilde{r}^2 + \frac{1}{4} \tilde{r}^2 \left(d\Omega_2^2 + \left(\frac{1}{W} d\tilde{x} - \cos \theta d\phi \right)^2 \right) \right]. \quad (4.5)$$

This metric describes a three-charge black hole placed in an orbifold space R^4/Z_W . The Bekenstein-Hawking entropy is

$$S_5 = 2\pi \sqrt{nwNW}. \quad (4.6)$$

The special case $W = 1$ yields the five-dimensional black hole in flat space (1.3).

The 4D-5D connection described above leads to an explicit construction of the near-horizon microstates of the 5D black holes (4.5). Since the modulus \tilde{R} is, as we have seen, a fixed scalar, the near-horizon geometry reduces to (3.4) for any value of \tilde{R} . In particular, starting from (4.2), (4.4) and taking the limit

$$\alpha' \rightarrow 0; \quad \frac{\tilde{r}^2}{\alpha'^{3/2}}, \frac{R}{\sqrt{\alpha'}}, \frac{V_{T^2}}{\alpha'}, \frac{V_{\tilde{T}^2}}{\alpha'} \text{ fixed} \quad (4.7)$$

and making similar coordinate changes as before, we obtain precisely the same near-horizon geometry (3.4) as in frame B. Hence the construction of the near-horizon microstates can be taken over from section 3.4. They are again given by horizon-wrapping KK-monopoles, whose moduli space dynamics contains a magnetic field (3.18). As mentioned above, we are particularly interested in the case $W = 1$ describing the D1-D5-P black hole in flat space. The counting of lowest Landau degeneracies involves solving the harmonic equation (2.15) on a product of two tori with magnetic fluxes w and N and a two-torus with one unit of magnetic flux coming from the Hopf bundle on S^2 . The construction of the harmonic forms in section 2.3 can be applied in this case and, proceeding as described there, the microscopic counting reproduces the Bekenstein-Hawking entropy (1.4).

We end with some further remarks:

- We should remark that the near-horizon scaling limit (4.7) differs from the one that is standard from the point of view of AdS_3/CFT_2 duality in that we are treating the S^1 radius R on the same footing as the other compact coordinates, focusing on energies small compared to $1/R$. The AdS_3/CFT_2 scaling limit would instead keep fixed \tilde{r}/α' , R , V_{T^2}/α' and $V_{\tilde{T}^2}/\alpha'$. From that point of view, the limit we have taken can be seen as an additional ‘very near horizon limit’ as described in [27].
- The calculation above is valid for a black hole ‘mostly made up out of D1-branes’ where the D1-charge n is parametrically larger than the other charges. From (3.6) we see that the supergravity description is reliable in this regime provided that $gw \gg 1$.

- In the S-dual picture, where the black hole consists of wrapped fundamental strings and NS5-branes and momentum, the relevant probe configurations are again wrapped KK monopoles, this time carrying induced fundamental string charge by turning on momentum conjugate to $\omega^{(0)}$.
- One can also find the relevant probe configurations when other charges are large by dualizing the relevant configurations in frame A. For large D5-charge w the probe solution is again a KK-monopole, this time wrapped on $\tilde{S}^3/Z_W \times \tilde{T}^2$. For large momentum N , one finds a D5-brane on \tilde{S}^3/Z_W and \tilde{T}^2 with momentum along S^1 , which can be interpreted as a giant graviton.

5. Discussion and outlook

In this paper, we have used U-duality and the 4D-5D connection to construct microstate probe solutions in the near-horizon geometry of the D1-D5-P black hole. The relevant configurations are bound states of D1-branes that have expanded through the Myers effect to form a Kaluza-Klein monopole wrapping the black hole horizon. They are static with respect to the time coordinate adapted to the L_0 generator of the ‘left-moving’ $SL(2, R)_L$, and hence are expected to correspond to bound states rather than fragmentation modes of the system [29]. It would be interesting to study in more detail the superconformal quantum mechanics describing the low-energy dynamics of the probes in the case $W = 1$, where the right-moving $U(1)$ symmetry is enhanced to $SO(3)_R$. In [7], a refined version of the near-horizon microstate approach was proposed, where it was argued that the probe brane quantum mechanics arises from the moduli space quantization of a multicentered solution. It would be of interest to study the the analogous ‘deconstructed’ black hole solutions and their moduli space in the case of the five-dimensional D1-D5-P black hole. As in [7], this is likely to provide a natural explanation for the fact that the probe branes are static with respect to the specific time coordinate τ .

The fact that the relevant near-horizon probes are KK-monopoles is also interesting in itself. In the approach advocated by Lunin and Mathur [12], the nonsingular microstate geometries in the D1-D5 system are due to the expansion of D1 and D5 branes into KK monopole supertubes [28]. It will be interesting to see if and how both approaches are related.

The probe solutions were constructed in the near-horizon geometry of the black hole which includes a quotient of AdS_3 (with the geometry of a BTZ black hole) and should be viewed as an averaged geometry describing an ensemble of microscopic states, corresponding to a density matrix in the dual CFT [30]. The probe solutions we have considered could be seen as adding black hole ‘hair’ to this averaged geometry. The states in the ensemble we considered are characterized by microscopic quantum numbers consisting of the wrapped KK-monopole charge and the angular momentum quantum numbers labelling the lowest Landau level groundstates. It would be interesting to identify these states within the known ensemble of microstates in the dual CFT. Such a comparison is obscured by the fact that the averaged near-horizon geometry has less symmetry than the dual CFT because

of the quotienting of AdS_3 . It could therefore be interesting to study the limit where the momentum N is much larger than the other charges where, as one can see from (3.4), the AdS_3 symmetries are approximately restored. As we remarked in section 3.4, the relevant probe solutions in this regime are a kind of giant gravitons. It would also be interesting to study the relation of such solutions to other giant graviton configurations constructed recently in [31, 32].

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